IOPscience iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Superconductivity from D3/D7: holographic pion superfluid

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

JHEP11(2009)070

(http://iopscience.iop.org/1126-6708/2009/11/070)

The Table of Contents and more related content is available

Download details:
IP Address: 80.92.225.132
The article was downloaded on 01/04/2010 at 13:31

Please note that terms and conditions apply.

Published by IOP Publishing for SISSA

Received: *January 21, 2009*
Revised: *September 23, 2009*
Accepted: *October 27, 2009*
Published: *November 16, 2009*

Superconductivity from D3/D7: holographic pion superfluid

Pallab Basu, Jianyang He, Anindya Mukherjee and Hsien-Hang Shieh

Department of Physics and Astronomy, University of British Columbia, 6224 Agricultural Road, Vancouver, B.C. V6T 1Z1, Canada

E-mail: pallab@phas.ubc.ca, jyhe@phas.ubc.ca, anindya@phas.ubc.ca, shieh@phas.ubc.ca

Abstract: We show that a D3/D7 system (in the limit of zero quark mass) at finite isospin chemical potential goes through a superconductor (superfluid) like phase transition. This is similar to a flavored superfluid phase studied in the QCD literature, where mesonic operators condense. We have studied the frequency dependent conductivity of the condensate and found a delta function peak in the zero frequency limit. This is an example of superconductivity in a string theory context. Consequently we have found a superfluid/supercurrent type solution and studied the associated phase diagram. The superconducting transition changes from second order to first order at a critical superfluid velocity. We have studied various properties of the superconducting system like superfluid density, energy gap, second sound etc. We investigate the possibility of the isospin chemical potential modifying the embedding of the flavor branes by checking whether the transverse scalars also condense at low temperatures. This however does not seem to be the case.

Keywords: Intersecting branes models, Gauge-gravity correspondence

© SISSA 2009

doi:10.1088/1126-6708/2009/11/070

JHEP11(2009)070

Contents

1 Introduction 1

2 General setup 2

3 Phase diagram 6
3.1 Speed of second sound 6

4 Frequency response 8

5 Effect of stationary isospin current 9

6 What condenses and what doesn't 10

7 Conclusions 12

A Transverse scalars and gauge fields on S_3 13

1 Introduction

The phase structure of Quantum Chromodynamics (QCD) and similar theories at finite values of the isospin and baryon chemical potentials is an interesting arena. At high enough isospin density the color singlet mesonic flavor degrees of freedom (e.g. pions) may go through a Bose-Einstein condensation. The physical motivation to study such a pion superfluid formed at high isospin density is related to the investigation of neutron stars, isospin asymmetric nuclear matter and heavy ion collisions at intermediate energies. Unfortunately this set of problems is difficult to tackle numerically due to the complex nature of the action. Various approaches including lattice simulations are used to investigate the nature of the QCD phase diagram at finite isospin chemical potential and the existence of a superfluid like state is argued [1–6].

One way to investigate various aspects of gauge theories is to use the gauge gravity duality [7] i.e., study a supergravity/tree level string theory to learn about large-N gauge theories. Although such examples do not include QCD or even pure Yang-Mills (YM) theory yet, many qualitatively similar models have been constructed. In the idealized limit where the ratio of flavor and color degrees of freedom is small, one can introduce probe branes in the gravity background to study flavor physics [8]. In this scenario, the baryon/isospin chemical potential maps to the chemical potentials for various gauge fields living on the brane. The issue of baryon and isospin chemical potentials has been addressed in various type of brane systems [9–15]. One phenomenon which is relatively less discussed in string theory literature is flavor superconductivity. There are isospin charged bosonic

JHEP11(2009)070

states with mass of $O(1)$ (e.g pions in QCD), which may be thought of as strings which have endpoints on different flavor branes. Such a state may naturally condense as we turn on the isospin chemical potential. Here we discuss such a scenario in a D3/D7 system in the zero quark mass limit. We introduce a couple of coincident branes which end on a AdS_5 black hole. We turn on a chemical potential corresponding to the SU(2) isospin gauge field living on the world volume of the branes and study the resulting superconducting phase transition and various properties associated with it.

The plan of the paper is as follows. In section 2 we will set up the probe brane configuration and the equations of motion for the gauge fields. In sections 3 and 4, we establish the superconducting phase transition and study the frequency dependent conductivity which has a pole (corresponds to DC superconductivity) at zero frequency. The speed of second sound is also calculated. Section 5 is devoted to a study of the DC supercurrent and the phase diagram as a function of the temperature and the velocity of the supercurrent. We discuss in detail the possibility of other relevant adjoint scalar fields also condensing at low temperatures through a similar mechanism in section 6. In section 7, we conclude and point out possible future extensions to the project.

Note added. When our work was near completion, another paper [16] was posted which deals with similar questions. These authors have however considered the full DBI action and hence the resulting details are a little different.

2 General setup

Let us consider $AdS_5 \times S^5$ in Poincare co-ordinates,

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx_1^2 + dx_2^2 + dx_3^2) + L^2 d\Omega_5^2 \tag{2.1}$$

with

$$f(r) = \frac{r^2}{L^2} - \frac{M}{r^2}, \tag{2.2}$$

where L is the radius of the anti-de Sitter space and M is related to the mass of the black hole. In this paper we will adopt the convention $M = L = 1$. The temperature of the black hole (and also of the boundary field theory) is given by

$$T = \frac{1}{\pi}. \tag{2.3}$$

It is more convenient to analyze the system by making a coordinate transformation $z = 1/r$. The metric becomes:

$$ds^2 = -f(z)dt^2 + \frac{dz^2}{z^4 f(z)} + \frac{1}{z^2}(dx_1^2 + dx_2^2 + dx_3^2) + d\Omega_5^2 \tag{2.4}$$

with

$$f(z) = \frac{1}{z^2} - z^2. \tag{2.5}$$

JHEP11(2009)070

The horizon is now at $z = 1$, while the conformal boundary lives at $z = 0$. In this background we will introduce two (or more) co-incident D7-branes. For simplicity we will consider the zero quark mass embedding, where the brane fills the whole AdS^5 and wraps the maximal S^3 of S^5. In this limit the effective induced metric on the brane will just be the $AdS_5 \times S^3$ metric.

$$ds^2_{\text{brane}} = -f(z)dt^2 + \frac{dz^2}{z^4 f(z)} + \frac{1}{z^2}(dx_1^2 + dx_2^2 + dx_3^2) + d\Omega_3^2 \tag{2.6}$$

The effective action for the brane fields is the Born-Infeld action,

$$S_{\text{DBI}} = -T_7 \int d^8x \sqrt{G + 2\pi\alpha' F}. \tag{2.7}$$

In order to consider the system at finite isospin chemical potential, we will add a pair of $D7$ brane probes. In this case F is a U(2) field strength on the world volume of the probes. We will not focus on baryonic $\mathrm{U}(1)_B$ and only investigate terms containing SU(2) isospin gauge fields. The string states which have their endpoints on different branes are charged under the isospin SU(2). The exact form of the non-Abelian DBI action is unknown [17]. To proceed further we will expand the action to leading order in λ_{YM4} keeping only Yang Mills terms.[1] Such a simplification has also been employed in studying other aspects of holographic QCD such as the meson spectra and baryon masses[18–20]. Such an approximation will be more accurate in the limit where the non-Abelian field strengths are small. The effective action now takes the form

$$S_{\text{DBI}} = -T_7 \frac{(2\pi\alpha')^2}{g_s} \int d^8x \sqrt{-G} Tr F^2 \propto N_c \int d^8x \sqrt{-G} Tr F^2 \tag{2.8}$$

where we have scaled out $7+1$ dimensional Yang Mills coupling g_7 and

$$F^a = \partial A^a + \epsilon^{abc} A^b A^c \tag{2.9}$$

On a phenomenological level eq. (2.8) may be thought as an effective holographic model of flavor superconductivity.

The setup is very similar to that of [21–23], where the non-Abelian gauge field is shown to condense at low temperatures in an AdS black hole background. Due to the non-Abelian nature of the SU(2) symmetry, the τ^1 and τ^2 components of the gauge field are charged under the τ^3 component. Hence turning on a chemical potential for the τ^3 component

[1] The square root form of the DBI action was considered in [16]. Their main result that the existence of condensate and superconductivity (i.e. a pole of imaginary part of the conductivity at zero frequency) is similar. However they find out finer details of the frequency response like other poles. It should be noted that our approximation of neglecting all the higher order terms is not a controlled approximation, as the chemical potential μ ($\propto F^2$ in the unit of α') is large ($\mu = 4$) near the phase transition. Keeping the whole DBI action changes the exact quantitative results but order of the magnitudes of quantities do not seem to change much. For example we have checked that in our convention phase transition occurs at $\mu \approx 7$ for the DBI corrected action. The scalar condensates (see 6) forms at $\mu \gtrsim 15$, although at such a low temperature numerics is less reliable. Interestingly, near integral value $\mu \approx 7$ implies a possibility of finding an exact solution as in sec 3.

JHEP11(2009)070

of the gauge field may lead to a condensation of the other two. This mechanism is very similar to condensation of a U(1) charged scalar discussed in [24]. Turning on a chemical potential for the τ^3 component of the gauge field breaks SU(2) to $\mathrm{U}(1)_I$. A condensation of the τ^1 or τ^2 component of the gauge field further breaks the $\mathrm{U}(1)_I$ symmetry. It should be mentioned that unlike $\mathrm{U}(1)_B$ where all charged states are baryonic and have masses of order $O(N)$, this $\mathrm{U}(1)_I$ theory has charged states (mesons) with $O(1)$ masses and their condensation can naturally be studied in terms of the probe brane picture.

We start with the ansatz

$$A = A_t \tau^3 dt + B_{x_1} \tau^1 dx_1 \tag{2.10}$$

We will assume spatial homogeneity in the field theory directions and our fields will only have dependence on the radial coordinate. The equations of motion for the fields in this coordinate system are:

$$A_t'' - \frac{A_t'}{z} - 2\frac{B_{x_1}^2}{z^2 f} A_t = 0 \tag{2.11}$$

$$B_{x_1}'' + \left(\frac{f'}{f} + \frac{1}{z}\right) B_{x_1}' + \frac{1}{z^2 f}\left(A_t^2 \frac{B_{x_1}}{z^2 f}\right) = 0 \tag{2.12}$$

For regularity at the horizon we will have to set $A_t = 0$ at $z = 1$. Since we have a set of coupled equations, this will in turn give the following conditions at the horizon ($z = 1$).

$$B_{x_1}' = 0$$
$$A_t = 0$$

Examining the behavior of the fields near the boundary, we find

$$A_t \sim \mu - \rho z^2 + \ldots$$
$$B_{x_1} \sim M_x + W_x z^2 + \ldots$$

Using gauge/gravity duality, μ, ρ are mapped to the isospin chemical potential[2] and charge density in the dual field theory, respectively. W_x is mapped to the expectation value of a meson operator which condenses at low temperatures. We will set the non-normalizable mode M_x to zero. In what follows we first establish that the mesonic condensate forms below a critical temperature. We compute the time dependent conductivity by turning on a spatial component for the isospin current as a fluctuation.

$$A_{x_3} = X(z) e^{i\omega t} \tau^3 dx_3 \tag{2.13}$$

[2]It is known that such a system at finite isospin chemical potential and zero temperature is unstable due to runaway Higgs VEV as the zero VEV configuration becomes a local maximum of the effective potential [25]. What happens at finite temperature is not completely clear. However one may imagine a system where Higgs VEV is artificially fixed to zero. We would like to thank Nick Evans for pointing this out. Alternatively one may consider a supersymmetric D3/D5 system which does not have such instabilities. The resulting equations of motion are almost the same as in the case of a D3/D7 system.

JHEP11(2009)070

The equation of motion for A_{x_3} is

$$X'' + \left(\frac{1}{z} + \frac{f'}{f}\right)X' - \; B_{x_1}^2 \frac{X}{z^2 f} + \frac{\omega^2 X}{z^4 f^2} = 0 \tag{2.14}$$

We will choose infalling boundary condition at the horizon $A_{x_3} \propto (z-1)^{-i\omega/4}$. Asymptotically,

$$X \sim S_x + J_x z^2 + ... \tag{2.15}$$

J_x corresponds to the isospin current, while S_x gives the dual current source (superfluid velocity). The conductivity[3] is given by

$$\sigma = Re\left[\frac{J_x}{i\omega S_x}\right] \tag{2.16}$$

X can be normalized to one at the horizon. As we will see, the conductivity has a pole at $\omega = 0$. This suggests that there is a DC supercurrent solution. To find such a solution we solve the following set of coupled time independent equations [27, 28]. Note here the effects of $A_{x_3} = X(z)$ on the other components of the gauge fields are taken into account, so this is not a fluctuation around the condensate formed by B_{x_1}. In this case the field A_{x_3} has the form $A_{x_3} = X(z)\tau^3 dx_3$.

$$\begin{aligned} A_t'' - \frac{A_t'}{z} - \frac{B_{x_1}^2}{z^2 f} A_t &= 0 \\ B_{x_1}'' + \left(\frac{f}{f'} + \frac{1}{z}\right) B_{x_1}' + \frac{1}{z^2 f}\left(A_t^2 \frac{B_{x_1}}{z^2 f} - B_{x_1}^3 - X^2 B_{x_1}\right) &= 0 \\ X'' + \left(\frac{1}{z} + \frac{f'}{f}\right)X' - \; B_{x_1}^2 \frac{X}{z^2 f} &= 0 \end{aligned} \tag{2.17}$$

For the regularity of A_{x_3} at the horizon,

$$X' = - \; B_{x_1}^2 \frac{X}{4}\bigg|_{z=1} \tag{2.18}$$

As we will see shortly, the system eq. (2.17) reveals a rich phase structure as the boundary value of A_{x_3} is tuned.

The convenient physical parameters for us are $(\frac{T}{\mu}, \frac{\omega}{\mu}, \frac{S_x}{\mu}, \frac{\sqrt[3]{W_x}}{\mu})$, or$(\frac{T}{\mu}, \frac{\omega}{\mu}, \frac{\sqrt[3]{J_x}}{\mu}, \frac{\sqrt[3]{W_x}}{\mu})$. We will use $\frac{\sqrt[3]{W_x}}{\mu}$is an order parameter and plot it as a function of $(\frac{T}{\mu}, \frac{S_x}{\mu})$. In practice, we choose to keep the temperature fixed and vary μ in this paper. The components of gauge fields on the three-sphere may also condense through a similar mechanism. We will examine these cases in the appendix. An important question is whether the isospin chemical potential would modify the embedding of the flavor branes. We will leave a detailed analysis of this for a future project. For now we will just check whether the transverse scalars also condense at low temperatures (section 6).

[3]There is a logarithmic correction to the conductivity in five dimensions [26], under which $\sigma \to \sigma + \frac{i\omega}{2}$. However such a term depends on the choice of renormalization and physical quantities like mass gap etc do not depend on it.

JHEP11(2009)070

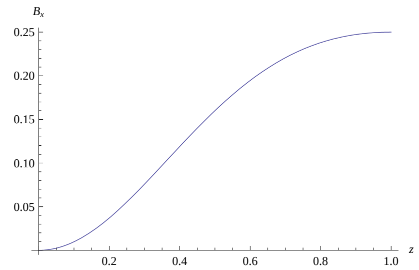


Figure 1. Plot of the zero mode at $\mu = 4$

3 Phase diagram

At high temperatures (or, equivalently, small values of μ) there is only one set of solutions to eq. (2.12) given by

$$\begin{aligned} A_t &= \mu(1 - z^2) \\ B_{x_1} &= 0 \end{aligned} \tag{3.1}$$

This should be interpreted as an isospin-charged black hole, where gauge fields are confined to the D7 brane. The dual gauge theory interpretation is a deconfined plasma with non-zero isospin charge. As we increase μ the effective mass of B_{x_1} in eq. (2.12) decreases and B_{x_1} develops a zero mode at $\mu = \mu_c = 4$. The existence of this zero mode can be analytically demonstrated. Substituting A_t from eq. (3.1) into the second of eq. (2.12) we get (this small fluctuation analysis does not depend on any possible cubic terms and is therefore true for other scalar field ansatzes considered later)

$$B''_{x_1} + \left(\frac{f}{f'} + \frac{1}{z}\right) B'_{x_1} + \left(\frac{\mu^2(1-z^2)^2}{z^4 f^2}\right) B_{x_1} = 0 \tag{3.2}$$

The above equation has an analytic solution for $\mu = 4$ given by

$$B_{x_1}(z) = \frac{z^2}{(1+z^2)^2} \tag{3.3}$$

The plot of this zero mode is shown in figure 3. Any further increment of μ leads to a condensation of B_{x_1}. Hence for $1/\mu < 0.25$ the solution develops a new branch with a non-zero value of B_{x_1}. Such a solution can be numerically constructed. The associated transition seems to be of second order from our numerics. In figure 2 we show a plot of the condensate with $\frac{1}{\mu}$ (a plot of the corresponding free energy is provided in section 6).

At low temperatures we find that the condensate levels off at $W_x/\mu^3 \approx 0.26$. In terms of the critical temperature T_c the condensate strength can be expressed as $W_x \approx 0.264^3\pi^3 T_c^3 \approx 515.94 T_c^3$ or $W_x^{\frac{1}{3}} \approx 8.01 T_c$.

3.1 Speed of second sound

The boundary field theory which is dual to the D3/D7 system in $AdS_5 \times S^5$ behaves like a superfluid below the critical temperature. Superfluids are known to exhibit modes known

JHEP11(2009)070

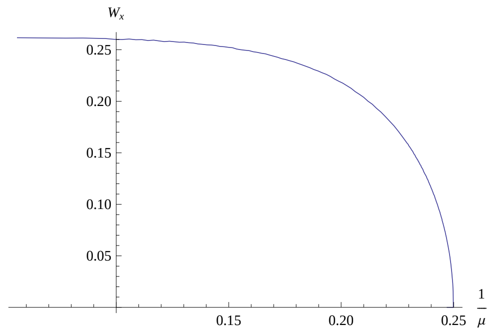


Figure 2. Plot of the condensate with $1/\mu$.

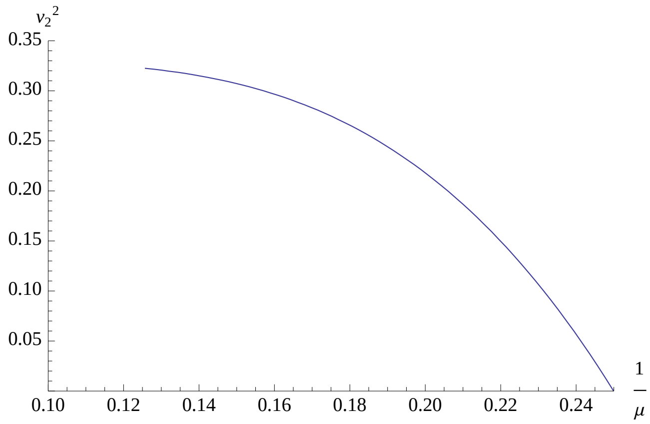


Figure 3. Speed of second sound as a function of $1/\mu$.

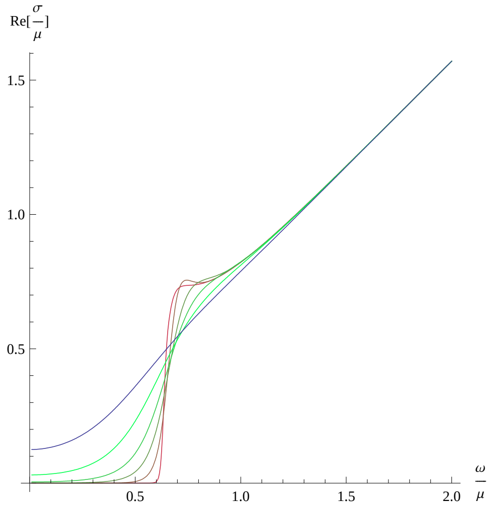
JHEP11(2009)070

as second sound which are basically temperature waves propagating through the fluid. For a hydrodynamic discussion see [28], where this was computed at zero superfluid velocity for the Abelian Higgs model on AdS_4. We also compute the speed of second sound in our case. The superfluid velocity now corresponds to S_x/μ, which we set to zero for this computation. The main relation is (see eq. (18) of [28]):

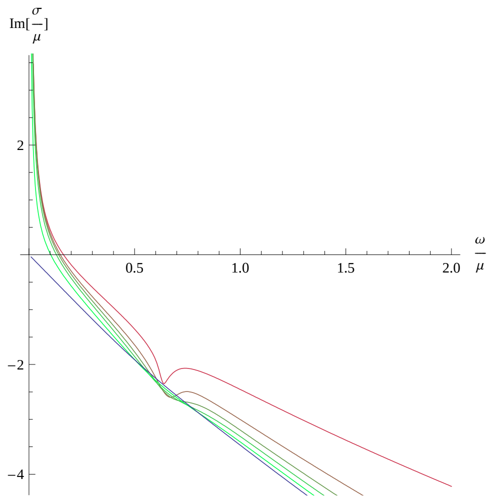
$$v_2^2 = \frac{\rho_s}{\mu \frac{\partial^2 P}{\partial \mu^2}}, \tag{3.4}$$

where ρ_s is the density of the superfluid component and P is the pressure. The pressure can be expressed in terms of the total fluid density ρ by using the equation of state of a perfect fluid $P = \mu\rho/(d-1)$ where $d = 4$ is the dimension in the fluid (boundary) theory.[4] Using this it is fairly simple to compute v_2^2. We present the result in figure 3, where we plot v_2^2 as a function of $1/\mu$. At high values of μ, v_2^2 approaches a limiting value $v_2^2 \approx 0.32$.

[4]The perfect fluid approximation is valid here since we are not considering any backreaction due to the metric. Hence there is no viscosity correction which originates from fluctuations of the metric.



(a) Plot of real part of $\frac{\sigma}{\mu}$



(b) Plot of imaginary part of $\frac{\sigma}{\mu}$

Figure 4. Plot of the real part of $\frac{\sigma}{\mu}$ with $\mu = 7.57\mu_c, 2.52\mu_c, 1.71\mu_c, 1.37\mu_c, 1.13\mu_c$ (gradually from red to green curves). The blue curve is for the exact frequency response at $\mu = \mu_c = 4$.

4 Frequency response

In this section we will study the frequency dependent conductivity of the spatial component of the isospin current eq. (2.13). In the absence of a condensate of B_{x_1} the frequency response can be exactly solved for [26], and is expressed in terms of digamma functions. In the presence of a condensate of the B_{x_1} field an analytic solution is difficult, but we can still numerically calculate the conductivity using eq. (2.16). First, we solve eq. (2.12) to determine the condensate. This solution is then taken as a fixed background on which we solve eq. (2.14) for the conductivity σ. In figure 4 we plot σ as a function of the frequency ω for various values of the parameter μ. We find that $Im[\sigma(\omega)] \sim \frac{n_s}{\omega}$ as $\omega \to 0$, where n_s is the superfluid density. Figure 5 shows a plot of n_s with $1/\mu$. Near $\mu = \mu_c$, n_s becomes proportional to $\mu - \mu_c$, vanishing at $\mu = \mu_c$. Fitting a linear function near the critical point we get $n_s \propto 0.1\mu_c^2(\mu - \mu_c)$.

The pole at $\omega = 0$ for $Im[\sigma(\omega)]$ implies $Re[\sigma(0)] \sim \pi n_s \delta(\omega)$ + terms regular in ω. This delta function singularity of the real part of sigma is not captured in the numerics directly. However this corresponds to superconductivity/superfluidity and consequently we can find a supercurrent/superfluid solution (see Sec 5). Unlike [16] we do not get any low temperature resonances in the conductivity. Our result is more similar to the zero mass Abelian-Higgs system presented in [26].[5]

As $\int Re[\sigma] d\sigma$ is a temperature invariant quantity, the delta function at $\omega = 0$ is compensated by a dip in $Re[\sigma]$ at low frequencies. The dip becomes more prominent as we lower the temperature (i.e., increase μ). It is clear from the plot (figure 4) that at low temperatures (large μ) $Re[\sigma] \to 0$. In fact it is expected that at low temperatures

[5] It seems that in an Abelian-Higgs system in AdS_5 resonances occur near the conformal mass [29].

JHEP11(2009)070

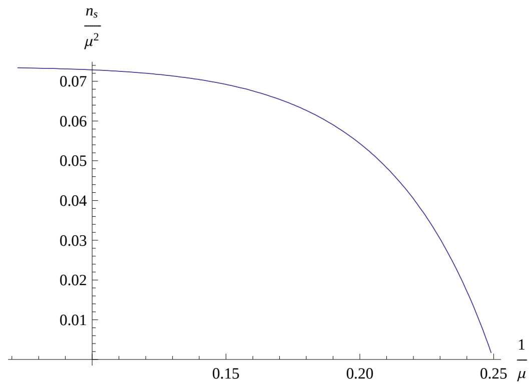


Figure 5. Plot of superfluid density with $1/\mu$.

$Re[\sigma] \sim \exp(-\frac{\Delta_g}{T})$, where Δ_g is the energy gap of the system. Also looking at the zero temperature limit of the real part of the conductivity we see that $Re[\sigma] = 0$ for $\omega \leq \Delta_p$. Δ_p is similar to the energy of a "Cooper pair". The ratio $n_g = \frac{\Delta_p}{\Delta_g}$ gives important information about the nature of the condensate. From our numerics we calculate

$$n_g \approx 1.2 \tag{4.1}$$

At high frequencies $Re[\sigma]$ computed at different temperatures (below T_c) approaches the zero condensate value.

5 Effect of stationary isospin current

In this section we investigate the effects of turning on a finite time-independent isospin X field (recall $A^a_\mu(r) = A_t(r)\tau^3 dt + B_{x_1}(r)\tau^1 dx_1 + X(r)\tau^3 dx_3$). We tune the boundary value of X, S_x/μ at a fixed isospin chemical potential μ to investigate the phase structure. At high enough μ and in absence of X the gauge component B_{x_1} condenses. As we can see from eq. (2.17), the effect of X is to increase the effective mass of the B_{x_1} field. This will cause the B_{x_1} condensate to weaken with increasing S_x/μ. Indeed, this happens, as we can see from figure 6. Here we plot the condensate strength as a function of the isospin current source S_x/μ for different chemical potentials μ. μ increases from left to right. For strong enough S_x/μ (above a critical value) there is a phase transition to the normal (non-superfluid) state. The order of this phase transition seems to be μ dependent. For high μ (compared to μ_c) the phase transition is first order, i.e. the system discontinuously jumps to the normal state above the critical current velocity S_x/μ. For μ close to μ_c the transition becomes second order. The order of the transition changes near $\mu_{sp} = 1.4\mu_c$. Note that in each case the condensate approaches a limiting value at low values of S_x/μ, and this limiting value decreases with decreasing μ.

In order to properly see the transition from first to second order, we also plot the difference in free energies[6] of the normal and superconducting branches. In figure 7 the left

[6]Free energy is calculated from the action eq. (2.7). The value of the Lorentzain action itself is the

JHEP11(2009)070

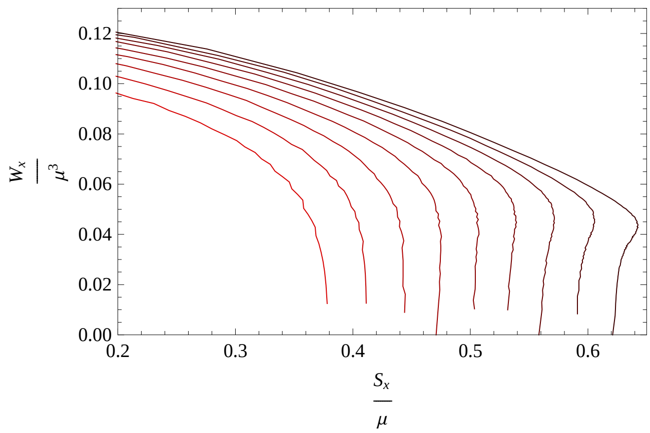


Figure 6. Plot of W_x/μ^3 as a function of S_x/μ at different values of μ. The values of μ range from $1.24\mu_c$ to $1.97\mu_c$ from left to right.

hand plot is for $\mu = 1.97\mu_c$ while the right hand plot is for $\mu = 1.24\mu_c$. We see the typical swallow tail shape indicating the abrupt change in dominance from the normal to the superconducting branch at low values of S_x/μ. We can understand this curve figure 7(a) in the following manner. As S_x/μ is lowered from above the critical value $S_{x,c}/\mu$, at first there is only a normal branch (I). At some value $S_{x,N}/\mu = 0.571$ two new branches are nucleated: one of these is stable (II), while the other one is unstable (III). The stable branch starts out with a higher free energy than the normal branch, but as S_x/μ is lowered further these two branches intersect at $S_{x,c}/\mu = 0.568$, where the branch (II) has the same free energy as branch (I). This is the first order phase transition point below which the system jumps to the superconducting branch (II) which now has lower free energy. At lower values of the chemical potential μ shown in figure 7(b) the transition is continuous between branches (I) and (II) at $S_{x,c}/\mu = 0.038$.

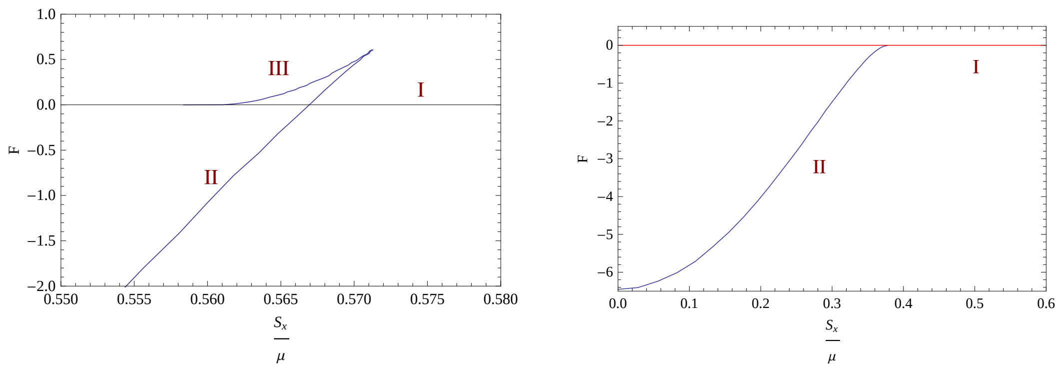
6 What condenses and what doesn't

There are various type of bosonic fields living on the branes which are charged under A^3_μ and may condense as we turn on a chemical potential for A^3_μ. These states come from strings ending on different branes. This includes gauge fields proportional to τ^1 and τ^2. Together with A^3, the trio corresponds to a vector/isovector "meson"[7] in the boundary theory. In this paper we consider an ansatz like eq. (2.10), but more generally one may consider $A = A_t\tau^3 dt + B_{x_1}\tau^1 dx_1 + C_x\tau^2 dx_2$. In this case the action contains a term proportional to

free energy. It should be noted that our evaluation of free energy is primarily numerical and there may be subtelty in the discussion of the phase transition. A better analytic understanding with possibly the full DBI action will be interesting. There may also be subtelty associated with boundary terms and gravity back reaction may be important in some situation. We thank refree for pointing this out.

[7]As $m_q = 0$ to begin with in our case, there is no stable meson. We only have quasinormal modes corresponding to various brane fields [30]. The dual interpretation of such modes are mesons decaying in the gauge theory plasma.

JHEP11(2009)070



(a) $\mu = 1.97$ (b) $\mu = 1.24$

Figure 7. Plot of the difference in free energies (F) of the normal and superconducting branches as a function of S_x/μ.

$B_{x_1}^2 C_x^2$ which describes a repulsive interaction between these two fields, so one may expect that both do not condense together. Also, an ansatz of the form $B_{x_1}(\tau^1 dx_1 + \tau^2 dx_2)$ [21] will generally have a higher free energy than what we have chosen.

Another possibility is the condensation of transverse scalar fields which exist on the brane. Brane fields are invariant under only a SO(4) $\times$ SO(2) subgroup of the full R-symmetry group SO(6) of S^5. A pair of transverse scalar fields correspond to one SO(2)-charged isovector scalar in the boundary theory. The Born-Infeld type effective action for such scalars is given in the appendix. We choose a general ansatz suitable for our case,

$$\Phi_1 = \phi_1 \tau^1, \quad \Phi_2 = \phi_2 \tau^2, \tag{6.1}$$

The EOM's are,

$$\phi_i'' + \left(-\frac{1}{z} + \frac{f'}{f} \right) \phi_i' + \frac{1}{z^4 f} \left(\frac{A_t^2}{f} - \phi_j^2 \right) \phi_i = 0, \tag{6.2}$$

where $i, j = 1, 2$ and $i \neq j$. From the discussion in the previous paragraph it is clear that due to the repulsive interaction term, a preferred configuration will have one of the two ϕ_i's turned off. One may try to find when such a field becomes unstable in a fixed A_t background. From our numerics we find out that for the solution eq. (3.2) such a thing happens at $\mu_c^s \approx 6.57 = 1.64\mu_c$ and greater than our $\mu_c = 4$ value for B_{x_1} field. Hence when we gradually decrease the temperature of our system, B_{x_1} condenses before any scalar degree of freedom. One may further ask wheather the resulting superconducting phase with a B_{x_1} condensate has an instability towards ϕ fluctuations. Our numerics answer this question negatively. It seems that some type of "blocking" mechanism stops further condensation of more fields. However that does not rule out the possibility of a first order transition between B_{x_1} condensed phase and ϕ condensed phase. We plot the free energy of both the phases to investigate a possible first order transition in figure 8.

It seems that the $B_{x_1} \neq 0$ phase always dominates. However our numerics is not very reliable for the parameter range $\mu > 10\mu_c$. Also, we did not exhaustively search for the possibility of various mixed phases. The case of gauge fields with S^5 indices is

JHEP11(2009)070

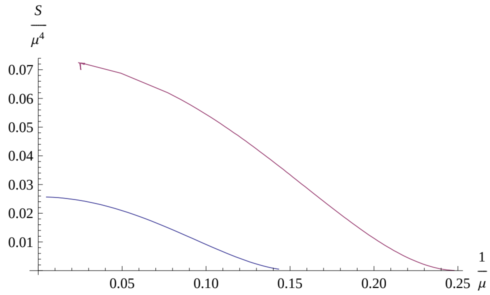


Figure 8. Plot of action for phases $B_{x_1} \neq 0$ (upper curve) and $\phi \neq 0$ (lower curve).

similar to that of the scalar fields (see appendix), at least for small fluctuations. Hence the blocking mechanism discussed above will work for them too and these fields will not also condense. However we have not investigated the possible phases and possibility of a first order transition in great detail for these fields.

7 Conclusions

In this paper we explored a model which provides a realization of holographic superconductivity/superfluidity in a string theory setup. We have studied a couple of probe D7 branes in an $AdS_5 \times S^5$ background. We have introduced a finite isospin chemical potential (i.e. potential for some of the world volume gauge fields) and have found the existence of a flavored superconducting state at high enough values of this chemical potential. We have studied the frequency dependent conductivity and have found a delta function pole in the zero frequency limit. This indicates a superconductor-like phase. Consequently we have found a superfluid/supercurrent type solution and have studied the resulting phase diagram. The superconducting transition changes from second order to first order at a critical a superfluid velocity. The holographic dual of such a string theory system is large N_c, $\mathcal{N} = 4$ supersymmetric gauge theory with $N_f \ll N_c$. In a dual gauge theory such a superconducting state is characterized by mesonic condensates. We have studied various properties of the superconducting system like energy gap, second sound etc.

In this paper we have discussed the possibility of a first order transition between various possible condensate. It is also important to check whether the isospin chemical potential modifies the embedding of the flavor branes and whether the transverse scalars actually condense. In our case they do not, but for some other setup they might. It would be interesting to address such questions in more detail.

In QCD, mesonic condensates (pion superfluids) have been argued to exist at finite isospin chemical potential. A natural extension of our work will be to study more realistic holographic models like [14, 18] or AdS/QCD like theories. It would be interesting to study such phenomena in cases where the fundamental degrees of freedom are fermionic. Here we focused on the zero quark mass case, which may be thought of as a high temperature limit. It would be natural to extend our work to the case of finite quark mass. Another interesting issue to investigate is flavor backreaction [31–33]. The mesonic operators which

JHEP11(2009)070

condense in our case are color singlets and do not lead to color superconductivity. It would be interesting to study some models with color superconductivity.

Acknowledgments

We would like to thank Mark van Raamsdonk and Moshe Rozali for helpful discussions and comments on our work. We thank the String Theory Group at UBC for their support and encouragement. PB and AM acknowledge support from the Natural Sciences and Engineering Research Council of Canada.

A Transverse scalars and gauge fields on S_3

Consider D7-branes embedded in an $AdS_5 \times S^5$ background,

$$ds^2 = -f(z)dt^2 + \frac{dz^2}{z^4 f(z)} + \frac{1}{z^2}(dx_1^2 + dx_2^2 + dx_3^2) + d\theta^2 + \sin^2\theta \left(d\varphi^2 + \sin^2\varphi \; d\Omega_3^2\right). \quad \text{(A.1)}$$

The induced metric is

$$ds_7^2 = -f(z)dt^2 + \frac{dz^2}{z^4 f(z)} + \frac{1}{z^2}(dx_1^2 + dx_2^2 + dx_3^2) + \sin^2\theta \sin^2\varphi \; d\Omega_3^2. \quad \text{(A.2)}$$

According to [17], the leading order DBI action of a D_p-brane is

$$S_p = -\frac{T_p(2\pi\alpha')^2}{4g_s} \int d^{p+1}\xi \; \sqrt{-G_{\text{ind}}} \text{Tr} \left[F_{ab}F^{ab} + 2\mathcal{D}_a\Phi_i\mathcal{D}^a\Phi^i + [\Phi_i, \Phi_j][\Phi^i, \Phi^j]\right], \quad \text{(A.3)}$$

where the scalars $\Phi^i \equiv X^i/(2\pi\alpha')$ are the transverse coordinates, and the covariant derivative is defined as $\mathcal{D}_a\Phi_i = \partial_a\Phi_i + [A_a, \Phi_i]$. For D_7-branes, one has two scalars Φ^i, $i = \theta, \varphi$.

We turn on a gauge field according to the ansatz $A = A_t\tau^3 dt$, where $\tau^a = \tau^a/2i$ with the commutation relations $[\tau^a, \tau^b] = \epsilon^{abc}\tau^c$. The scalars take the general form $\Phi^i = \Phi^i_a\tau^a$. The non-zero components of $\mathcal{D}_a\Phi_i$ are

$$\mathcal{D}_t\Phi_i = [A_t\tau^3, \Phi_i] = A_t(\Phi_{i,2}\tau^1 - \Phi_{i,1}\tau^2), \qquad \mathcal{D}_z\Phi_i = \partial_z\Phi_i = (\partial_z\Phi_{i,a})\tau^a. \quad \text{(A.4)}$$

The effective action can be written as

$$S_7 = -\frac{T_7(2\pi\alpha')^2}{4g_s} \int d^8\xi \; \frac{\sin^3\theta \sin^3\varphi}{z^5} \Bigg\{ \text{Tr}\left(F_{ab}F^{ab}\right) - g_{ii}\left[z^4 f(z)\sum_{l=1}^{3}(\partial_z\Phi^i_l)^2 - f(z)A_t^2((\Phi^i_1)^2 + (\Phi^i_2)^2)\right] - \frac{1}{2}\sin^2\theta\left[\sum_{l\neq m}^{1,2,3}(\Phi^\theta_l\Phi^\varphi_m - \Phi^\theta_m\Phi^\varphi_l)^2\right]\Bigg\}, \quad \text{(A.5)}$$

where g_{ii} are 10-dimensional metric components with $i = \theta$ or φ.

JHEP11(2009)070

Introducing $\Phi^i_\pm \equiv (\Phi^i_1 \pm i\Phi^i_2)/2$, we find the equations of motion for $(\Phi^i_3, \Phi^i_\pm)$,

$$\begin{aligned}
&g_{ii}\left[\frac{f(z)}{z}\partial_z^2\Phi^i_3 + \left(-\frac{f}{z^2}+\frac{f'(z)}{z}\right)\partial_z\Phi^i_3\right] = 2\frac{\sin^2\theta}{z^5}\left[\Phi^i_3((\Phi^j_-)^2+\Phi^j_+\Phi^j_-)-\Phi^j_3(\Phi^i_+\Phi^j_-+\Phi^i_-\Phi^j_+)\right],\\
&g_{ii}\left[\frac{f(z)}{z}\partial_z^2\Phi^i_+ + \left(-\frac{f}{z^2}+\frac{f'(z)}{z}\right)\partial_z\Phi^i_+\right]\\
&\qquad = \frac{1}{z^5}\left\{-\frac{g_{ii}}{f}A_t^2\Phi^i_+ + \sin^2\theta\left[\Phi^i_+\Phi^j_+\Phi^j_- + \Phi^i_-(-(\Phi^j_+)^2+(\Phi^j_3)^2) - \Phi^i_3\Phi^j_-\Phi^j_3\right]\right\}\\
&g_{ii}\left[\frac{f(z)}{z}\partial_z^2\Phi^i_- + \left(-\frac{f}{z^2}+\frac{f'(z)}{z}\right)\partial_z\Phi^i_-\right]\\
&\qquad = \frac{1}{z^5}\left\{-\frac{g_{ii}}{f}A_t^2\Phi^i_- + \sin^2\theta\left[\Phi^i_-\Phi^j_+\Phi^j_- + \Phi^i_+(-(\Phi^j_-)^2+(\Phi^j_3)^2) - \Phi^i_3\Phi^j_+\Phi^j_3\right]\right\},
\end{aligned} \tag{A.6}$$

where $i, j \in (\theta, \varphi)$ and $i \neq j$.

Consider fluctuations. One can set $\sin\theta = 1$, and thus $g_{ii} = 1$ in the above equations. If we limit our discussion to the ansatz

$$\Phi_1 = \phi_1\tau^1, \quad \Phi_2 = \phi_2\tau^2, \tag{A.7}$$

the the EOM's can be simplified

$$\phi_i'' + \left(-\frac{1}{z}+\frac{f'}{f}\right)\phi_i' + \frac{1}{z^4 f}\left(\frac{A_t^2}{f} - \phi_j^2\right)\phi_i = 0, \tag{A.8}$$

where $i, j = 1, 2$ and $i \neq j$.

For the simplest case, with $\Phi = \phi(z)(\tau^1 dx^1 + \tau^2 dx^2)$, the equation of motion is

$$\phi'' + \left(-\frac{1}{z}+\frac{f'}{f}\right)\phi' + \frac{1}{z^4 f}\left(\frac{A_t^2}{f}\phi - \phi^3\right) = 0. \tag{A.9}$$

If the gauge fields on the 3-sphere are turned on instead of the transverse scalars on the D7-branes, one would find a similar condensation phenomenon. In an ansatz

$$A \sim A_t(z)\tau^3 dt + A_\theta(z)\tau^1 d\theta, \tag{A.10}$$

the equations of motion are

$$\begin{aligned}
A_t'' - \frac{1}{z}A_t' - \frac{A_\theta^2}{z^4 f}A_t &= 0,\\
A_\theta'' + \left(-\frac{1}{z}+\frac{f'}{f}\right)A_\theta' + \frac{A_t^2}{z^4 f^2}A_\theta &= 0.
\end{aligned} \tag{A.11}$$

Or in another ansatz,

$$A \sim A_t\tau^3 dt + A_\theta(\tau^1 d\theta + \tau^2 \sin\theta d\varphi), \tag{A.12}$$

JHEP11(2009)070

where (θ, φ) are angle coordinates on 3-sphere of D7-brane worldvolume $AdS_5 \times S^3$. The EOM's turn out to be

$$\begin{aligned} A_t'' - \frac{1}{z}A_t' - \frac{2A_\theta^2}{z^4 f}A_t &= 0, \\ A_\theta'' + \left(-\frac{1}{z} + \frac{f'}{f}\right)A_\theta' + \frac{1}{z^4 f}\left(\frac{A_t^2}{f}A_\theta - A_\theta^3\right) &= 0. \end{aligned} \tag{A.13}$$

References

[1] K. Splittorff, D.T. Son and M.A. Stephanov, *QCD-like Theories at Finite Baryon and Isospin Density*, *Phys. Rev.* **D 64** (2001) 016003 [hep-ph/0012274] [SPIRES].

[2] D.T. Son and M.A. Stephanov, *QCD at finite isospin density: From pion to quark antiquark condensation*, *Phys. Atom. Nucl.* **64** (2001) 834 [hep-ph/0011365] [SPIRES].

[3] D.T. Son and M.A. Stephanov, *QCD at finite isospin density*, *Phys. Rev. Lett.* **86** (2001) 592 [hep-ph/0005225] [SPIRES].

[4] D. Toublan and J.B. Kogut, *Isospin chemical potential and the QCD phase diagram at nonzero temperature and baryon chemical potential*, *Phys. Lett.* **B 564** (2003) 212 [hep-ph/0301183] [SPIRES].

[5] J.B. Kogut and D.K. Sinclair, *Lattice QCD at finite isospin density at zero and finite temperature*, *Phys. Rev.* **D 66** (2002) 034505 [hep-lat/0202028] [SPIRES].

[6] L.-y. He, M. Jin and P.-f. Zhuang, *Pion superfluidity and meson properties at finite isospin density*, *Phys. Rev.* **D 71** (2005) 116001 [hep-ph/0503272] [SPIRES].

[7] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200] [SPIRES].

[8] A. Karch and E. Katz, *Adding flavor to AdS/CFT*, *JHEP* **06** (2002) 043 [hep-th/0205236] [SPIRES].

[9] M. Rozali, H.-H. Shieh, M. Van Raamsdonk and J. Wu, *Cold Nuclear Matter In Holographic QCD*, *JHEP* **01** (2008) 053 [arXiv:0708.1322] [SPIRES].

[10] D. Mateos, S. Matsuura, R.C. Myers and R.M. Thomson, *Holographic phase transitions at finite chemical potential*, *JHEP* **11** (2007) 085 [arXiv:0709.1225] [SPIRES].

[11] J. Erdmenger, M. Kaminski, P. Kerner and F. Rust, *Finite baryon and isospin chemical potential in AdS/CFT with flavor*, *JHEP* **11** (2008) 031 [arXiv:0807.2663] [SPIRES].

[12] S. Kobayashi, D. Mateos, S. Matsuura, R.C. Myers and R.M. Thomson, *Holographic phase transitions at finite baryon density*, *JHEP* **02** (2007) 016 [hep-th/0611099] [SPIRES].

[13] J. Erdmenger, M. Kaminski and F. Rust, *Holographic vector mesons from spectral functions at finite baryon or isospin density*, *Phys. Rev.* **D 77** (2008) 046005 [arXiv:0710.0334] [SPIRES].

[14] O. Aharony, K. Peeters, J. Sonnenschein and M. Zamaklar, *Rho meson condensation at finite isospin chemical potential in a holographic model for QCD*, *JHEP* **02** (2008) 071 [arXiv:0709.3948] [SPIRES].

JHEP11(2009)070

[15] K. Ghoroku, M. Ishihara and A. Nakamura, *D3/D7 holographic Gauge theory and Chemical potential*, *Phys. Rev.* **D 76** (2007) 124006 [arXiv:0708.3706] [SPIRES].

[16] M. Ammon, J. Erdmenger, M. Kaminski and P. Kerner, *Superconductivity from gauge/gravity duality with flavor*, *Phys. Lett.* **B 680** (2009) 516 [arXiv:0810.2316] [SPIRES].

[17] C.V. Johnson, *D-brane primer*, hep-th/0007170 [SPIRES].

[18] T. Sakai and S. Sugimoto, *More on a holographic dual of QCD*, *Prog. Theor. Phys.* **114** (2005) 1083 [hep-th/0507073] [SPIRES].

[19] K. Nawa, H. Suganuma and T. Kojo, *Baryons in Holographic QCD*, *Phys. Rev.* **D 75** (2007) 086003 [hep-th/0612187] [SPIRES].

[20] H. Hata, T. Sakai, S. Sugimoto and S. Yamato, *Baryons from instantons in holographic QCD*, hep-th/0701280 [SPIRES].

[21] S.S. Gubser, *Colorful horizons with charge in anti-de Sitter space*, *Phys. Rev. Lett.* **101** (2008) 191601 [arXiv:0803.3483] [SPIRES].

[22] M.M. Roberts and S.A. Hartnoll, *Pseudogap and time reversal breaking in a holographic superconductor*, *JHEP* **08** (2008) 035 [arXiv:0805.3898] [SPIRES].

[23] S.S. Gubser and S.S. Pufu, *The gravity dual of a p-wave superconductor*, *JHEP* **11** (2008) 033 [arXiv:0805.2960] [SPIRES].

[24] S.A. Hartnoll, C.P. Herzog and G.T. Horowitz, *Building a Holographic Superconductor*, *Phys. Rev. Lett.* **101** (2008) 031601 [arXiv:0803.3295] [SPIRES].

[25] R. Apreda, J. Erdmenger, N. Evans and Z. Guralnik, *Strong coupling effective Higgs potential and a first order thermal phase transition from AdS/CFT duality*, *Phys. Rev.* **D 71** (2005) 126002 [hep-th/0504151] [SPIRES].

[26] G.T. Horowitz and M.M. Roberts, *Holographic Superconductors with Various Condensates*, *Phys. Rev.* **D 78** (2008) 126008 [arXiv:0810.1077] [SPIRES].

[27] P. Basu, A. Mukherjee and H.-H. Shieh, *Supercurrent: Vector Hair for an AdS Black Hole*, *Phys. Rev.* **D 79** (2009) 045010 [arXiv:0809.4494] [SPIRES].

[28] C.P. Herzog, P.K. Kovtun and D.T. Son, *Holographic model of superfluidity*, *Phys. Rev.* **D 79** (2009) 066002 [arXiv:0809.4870] [SPIRES].

[29] P. Basu, J. He, A. Mukherjee and H.-H. Shieh, work in progrss.

[30] P.K. Kovtun and A.O. Starinets, *Quasinormal modes and holography*, *Phys. Rev.* **D 72** (2005) 086009 [hep-th/0506184] [SPIRES].

[31] R. Casero, C. Núñez and A. Paredes, *Towards the string dual of $N = 1$ SQCD-like theories*, *Phys. Rev.* **D 73** (2006) 086005 [hep-th/0602027] [SPIRES].

[32] S.-J. Sin, *Gravity Back-reaction to the Baryon Density for Bulk Filling Branes*, *JHEP* **10** (2007) 078 [arXiv:0707.2719] [SPIRES].

[33] P. Basu and A. Mukherjee, *Dissolved deconfinement: Phase Structure of large-N gauge theories with fundamental matter*, *Phys. Rev.* **D 78** (2008) 045012 [arXiv:0803.1880] [SPIRES].